

# **IMPROVED DIGITAL ALGORITHM FOR DISTANCE RELAY TAKING INTO ACCOUNT THE GROUNDING IMPEDANCE AT THE FAULT PLACE**

**Lj. Popović, JP "Elektrodistribucija-Beograd", Srbija i Crna Gora**

## **INTRODUCTION**

The towers of transmission lines are grounded throughout their footing electrodes that are mutually connected through a ground wire(s). Because of that, during the ground fault one part of a fault current is dissipated into the earth through the faulted tower, ground wire(s) and many others towers. At this the potential that appears at the fault place (faulted tower) can be significant (several kV). The algorithm for ground fault distance relay developed in the previously published paper completely ignores this fact (e.g. [1]). The first attempt of taking into account the grounding impedance at fault place is undertaken in [2]. However, the algorithm developed in this paper corresponds only to the situation without the ground fault current component coming from the opposite line end. Because of that applicability of this algorithm is limited on a relatively small number of practical cases. In this regard algorithm presented in this paper represents significant enlargement of the applicability scope of the algorithms taking into account the grounding impedance at the fault place.

This paper is a logical continuation of the former publications [3,4] and belongs to the methods that calculate the distance to the fault place by using fundamental frequency voltage and current from one terminal of the transmission line. The proposed algorithm is focused on a single phase-to-ground fault, a type of fault that occupies about 90% of all of the transmission line faults (e.g. [4]). Finally, the proposed algorithm significantly compensates the deficiency of the data on the relevant factors – the fault impedance and the fault current coming from the opposite line end. This is done by analyzing the ground fault current return paths and by using the fact that the ratio between the real and the imaginary part of the fault impedance varies along the whole line length in a very narrow range of values. As a result of this it is obtained the algorithm of better accuracy in comparing with earlier known [1,3]. This quality enables us for zone-1 setting of distance relays without causing coordination problems characteristic for relatively short transmission lines, as they are lines in high voltage distribution networks.

## **BASIC PROBLEM**

Let us assume that we have two parts of a power system with directly grounded neutral points and connected by a single circuit line. For a single phase-to-ground fault occurring anywhere along the line, the electrical circuits established during the fault may be schematically presented as in Fig. 1.

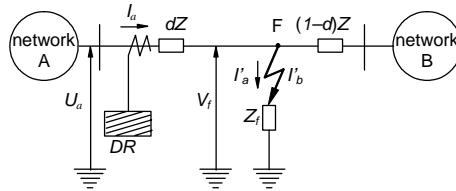


Fig. 1. Faulted power system

On the basis of the quantities measurable on the left line terminal (voltage  $U_a$  and current  $I_a$ ) it is necessary to find out the distance to the fault point ( $d$ ). If we assume for a moment that the fault impedance  $Z_f$  is negligible, the line impedance to the fault point  $dZ$  will be very easy to determine. As this impedance is proportional to the distance  $d$ , the determination of the fault position will be also easy. However, under practical conditions the fault impedance is not negligible and represents a very complex function of the distance to the fault location [3,4]. As a consequence of this fact, at the fault place a potential  $V_f$  appears with a value proportional to the total fault current ( $I'_a + I'_b$ ) for a certain value of the impedance  $Z_f$ . Since the fault current from the opposite end of the line  $I'_b$  contributes to the creation of this potential, the microprocessor-based relay DR sees an apparent impedance that is somewhat larger than the real impedance ( $dZ + Z_f$ ). This increase introduces a deviation in the measurement data ( $U_a$  and  $I_a$ ) that we will use for the fault distance determination. The deviation is more pronounced when the relative share of the current  $I'_b$  in the total fault current is larger.

The pre-fault (load) current  $I_p$  has a similar influence on the measured quantities. This current is not separately presented in the given circuit, but it is clear that this current represents the difference between the current through the faulted phase conductor,  $I_a$ , and the current through the fault place  $I'_a$ .

The above discussion can be summarized into the following. For a more accurate detection of the fault location it is necessary to eliminate the influence of the unknown (not available by measurement) quantities on the data obtained by measurement. This certainly means that our investigation should be focused on the unknown but relevant factors. In the circuit shown in Fig. 1 the fault impedance  $Z_f$  represents only one part of the loop impedance measured at the relay location, or in the local station. However, in practice this is an equivalent impedance of a very complex and spontaneously formed electrical circuit. Because of that, it is certain that more information about the fault impedance can be obtained only through a detailed investigation of this circuit.

During a ground fault a transmission line represents a very complex electrical circuit with a large number of conductively and inductively coupled elements. Towers of transmission lines are grounded through the footing electrodes and mutually connected by a ground wire(s). At the place of the faulted tower the ground fault current leaves the phase conductor. Its flow to the feeding sources continues through many different paths. Due to inductive coupling between the phase conductors and the ground wire(s), a part of this current circulates exclusively through metal paths (in the line, through the ground wire(s) and in the station, through the grounding connections.) The remaining part of the fault current returns conductively to the power system, through the earth via ground wire(s), through a large number of towers and through the grounding systems of all substations with grounded neutral point(s).

## APPARENT IMPEDANCE FOR THE RETURN PATHS OF GROUND-FAULT CURRENT

We will start our consideration by assuming that the substations at the line terminals are the only substations with grounded neutral point(s) in the whole power system. This means that the total ground fault current  $I_f$  returns to the power system only through the grounded neutral point(s) of these substations (A and B in Fig. 2). The real physical model of the transmission line under the conditions of a ground fault at an arbitrary tower is schematically presented in Fig. 2.

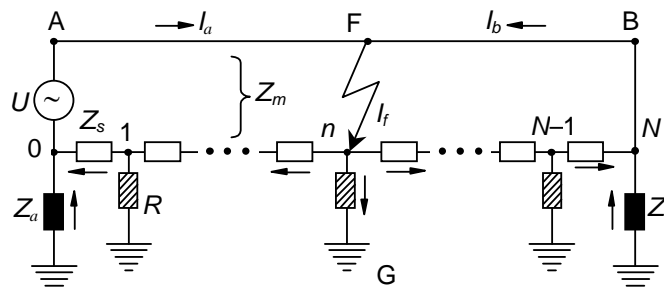


Fig. 2. Three-phase transmission line with a ground fault

The notation used in this circuit has the following meaning:

- $I_a(I_b)$  – part of the current  $I_f$  flowing left (right) from the fault place,
- $Z_a(Z_b)$  – impedance of the grounding system of the substation A (B) which does not include the grounding effects of the ground wire(s) of the line under consideration,
- $Z_s$  – self impedance of the ground wire(s), per span,
- $Z_m$  – mutual impedance between the ground wire(s) and the faulted phase conductor, per span,
- $R$  – average tower footing resistance,
- $n$  – number of spans to the fault location, counted from the substation A,
- $N$  – overall number of spans,
- $G$  – remote ground.

By forming the presented transmission line model, the following approximations and idealizations of the real physical model were used:

- phase and ground wire(s) impedances and their mutual impedances are identical to the values calculated on the basis of infinite transmission lines,
- towers footing resistances are mutually equal and any mutual interference to their own ground current is neglected,
- impedances of the ground wire(s) between the two towers are mutually equal.

The impedances  $Z_s$  and  $Z_m$  can be either calculated by using formulae based on Carson's theory of the ground fault current return path (e.g. [6]) or measured.

In order to simplify the problem we will consider only the elements representing the ground fault current return paths, as shown in Fig. 3.

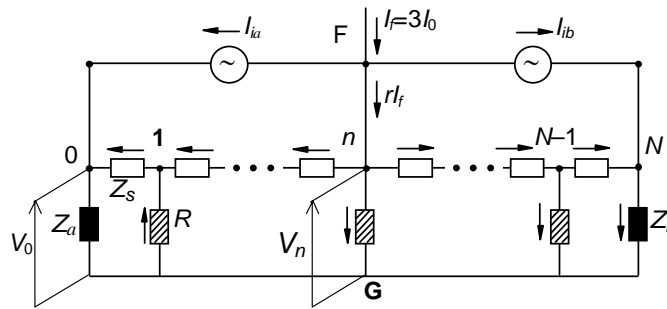


Fig. 3. Return paths of ground fault current

In this circuit the influence of the inductive coupling between the ground wire and the phase conductors is separately presented (e.g. [2]). The induced current and the corresponding so-called reduction factor of the line are determined by

$$I_{ia} = \frac{Z_m}{Z_s} I_a \quad (1)$$

$$I_{ib} = \frac{Z_m}{Z_s} I_b \quad (2)$$

$$r = \left( 1 - \frac{Z_m}{Z_s} \right) \quad (3)$$

Since a transmission line is usually transposed, the value of the reduction factor varies from tower to tower. These variations are limited between the values corresponding to the closest and the farthest position of phase conductor with respect to the ground wire(s).

Since we are interested only in the potential differences between the points  $n$  and  $0$  (or  $n$  and  $N$ ), the equivalent circuit represented in Fig. 3 can be reduced to the circuit presented in Fig. 4.

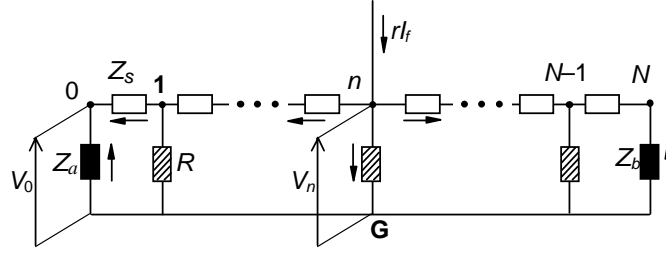


Fig. 4. All-conductive couplings on the ground-fault current return paths

Looking from the fault point F we discern two uniform lumped-parameter ladder circuits with a finite number of elements. In general case, the voltage and the current at the input end ( $V_0$  and  $I_0$ ) and the voltage and the current at the output end ( $V_n$  and  $I_n$ ) according to [2] are related by

$$\begin{aligned} V_0 &= \frac{k^{2n} + k}{k^n + k^{n+1}} V_n + \frac{(k^{2n} + 1)Z_\infty}{k^n + k^{n+1}} I_n \\ I_0 &= \frac{k^{2n} - k^2}{(k^n + k^{n+1})Z_\infty} V_n + \frac{k^{2n} + k}{k^n + k^{n+1}} I_n \end{aligned} \quad (4)$$

The parameter  $k$  is the current distribution factor at any node, assuming that the number of the nodes is infinite. This parameter is determined by

$$k = 1 + \frac{Z_\infty}{R} \quad (5)$$

The impedance  $Z_\infty$  represents the input impedance of the uniform ladder circuit with an infinite number of nodes. This impedance is given by

$$Z_\infty = \frac{Z_s}{2} + \sqrt{RZ_s + \frac{Z_s^2}{4}} \quad (6)$$

However, in our case the problem is such that it cannot be solved by direct application of these equations. Before that, the real electrical circuit shown in the previous Figure should be modified.

By using the superposition principle, the equivalent circuit shown in Fig. 3 can be substituted (with regards to the potentials appearing in the points 0, 1, 2, ...  $n$ ) by the equivalent circuit presented in Fig. 5.

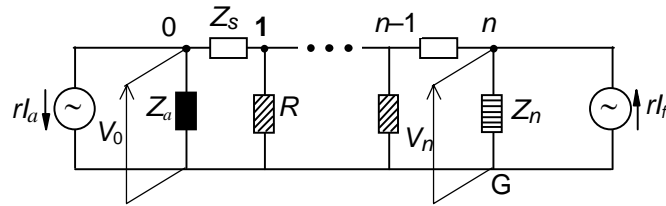


Fig. 5. Auxiliary equivalent circuit

The impedance  $Z_{nr}$  represents the grounding impedance of the transmission line ground wire(s) seen from the point F towards the station B. Its value depends on the fault place. In practical conditions the influence of the impedance  $Z_b$  on the value of the impedance  $Z_{nr}$  is so small that it can be neglected ( $Z_b \approx 0$ ). On the basis of this approximation and by using equations (4) we get

$$Z_{nr}(Z_b \approx 0) = \left( \frac{1}{R} + \frac{k^{2(N-n)} + k}{k^{2(N-n)} - 1} Z_\infty \right)^{-1} \approx \frac{k^{2(N-n)} - 1}{k^{2(N-n)} + k} Z_\infty \quad (7)$$

After the described modification, we have a circuit convenient for the application of the general equations of the uniform ladder circuit.

According to the equivalent circuit in Fig. 5, the potential in the point 0 created only by the current source  $rI_a$  is given by

$$V_0(rI_a) = -rZ_a I_a \quad (8)$$

On the basis of (8) and the equations (4), the potential at the point  $n$  created by the current sources  $rI_a$  is given by

$$V_n(rI_a) = \frac{-rZ_a I_a}{\frac{k^{2n} + k}{k^n + k^{n+1}} + \frac{k^{2n} + 1}{k^n + k^{n+1}} \frac{k^{2(N-n)} - 1}{k^{2(N-n)} + k}} \quad (9)$$

or, with certain approximations

$$V_n(rI_a) \approx \frac{-rZ_a I_a}{(0.5k^n + 0.5k^{-n}) \left( 1 + \frac{k^{2N} k^{-2n} - 1}{k^{2N} k^{-n} + k} \right)} \quad (10)$$

By using an analogous procedure, the potential at the point  $n$  created by the current sources  $rI_f$  is given by

$$V_n(rI_f) = rZ_n I_f \quad (11)$$

where the impedance  $Z_n$  represents the grounding impedance of the transmission line ground wire at the fault place. Using equations (8) and assuming that  $Z_a \approx 0$  and  $Z_b \approx 0$ , this impedance is given by

$$Z_n \approx \left[ \frac{2k^{2N} + (k-1)k^{2(N-n)} + (k-1)k^{2n} - 2k}{(k^{2N} - k^{2(N-n)} - k^{2n} + 1)Z_\infty} - \frac{1}{R} \right]^{-1} \quad (12)$$

When the potential  $V_n(rI_f)$  is defined by using the general equations of uniform ladder circuit (4), the potential  $V_0(rI_f)$  can be expressed as

$$V_0(rI_f) = \frac{\frac{(k^{2N} - k^{2n} - k^{2N} k^{-2n} + 1)Z_\infty}{2k^{2N} + (k-1)k^{2n} + (k-1)k^{2N} k^{-2n} - 2k}}{\frac{k^{2n} + k}{k^n + k^{n+1}} + \frac{k^{2n} + 1}{k^n + k^{n+1}} \frac{Z_\infty}{Z_a}} rI_f \quad (13)$$

By using the superposition principle, the real potentials at the points 0 and  $n$  are determined by

$$V_0 = V_0(rI_a) + V_0(rI_f) \quad (14)$$

$$V_n = V_n(rI_a) + V_n(rI_f) \quad (15)$$

Finally, we can express the voltage drop on the ground-fault current return paths as

$$U_{nr} = V_n(rI_a) + V_n(rI_f) - V_0(rI_a) - V_0(rI_f) \quad (16)$$

In practical conditions the relations between the considered quantities are such ( $Z_a \approx Z_b \ll Z_n$  and  $I_a \ll I_f$ ) that potentials  $V_n(rI_a)$ ,  $V_0(rI_a)$  and  $V_0(rI_f)$  can be disregarded. Thus we can write

$$U_{nr} \approx V_n(rI_f) = V_f = rI_f Z_n \quad (17)$$

At the same time, it means that instead of the impedance of the all of the ground fault current return paths, it is sufficient to consider only impedance  $Z_n$  (or  $Z_n \approx Z_f$ ).

When we have the expression (17), the apparent impedance for the ground fault current return paths according to Fig. 2 is given by

$$Z_{fa} = \frac{U_{nr}}{I_a} \approx \frac{V_f}{I_a} = r\gamma Z_f \quad (18)$$

where

$$\gamma = 1 + \frac{I_b - I_p}{I_a} \quad (19)$$

The coefficient  $\gamma$  represents a complex number with an imaginary part that can be disregarded in many practical situations. Its value along the line length varies in a very wide range of realistically possible values. On the basis of (19) it is interesting to note that the effects of the currents  $I_b$  and  $I_p$  (load current) depend on the direction of the current  $I_p$  (from A to B, or opposite) and accordingly may cancel or supplement each other. The current  $I_a$  represents the measured component of the fault current, but include the load current as well.

Many of the quantitative analyses already performed show that the reduction factor  $r$  represents a complex number with a negligible imaginary part. Its value varies along the line in a very narrow range of values [2] and can be determined by pre-calculation, or by pre-measurement. When the line is untransposed, this factor has the same value along the whole line length, but different for each of the phase conductors because of their different space positions in relation to the ground wire.

The variations of the effective value of the impedance  $Z_f$  along the line length are considered in [3,4], while in the paper [5] a quantitative analysis of the real and the imaginary part of this impedance is performed.

As a conclusion it can be said that the potential at the fault place  $V_f$  (Fig. 1) depends on the factors  $r$ ,  $\gamma$  and  $Z_f$ . Their values depend on the fault place and cannot be determined if the fault place is an unknown quantity.

## ALGORITHM DERIVATION

According to the circuit shown in Fig. 1, the measured voltage  $U_a$  is the sum of the voltage drop in the line to the fault point and the fault point potential  $V_f$ . By dividing the measured voltage  $U_a$  with the measured current  $I_a$ , according to (18), we obtain

$$Z_a = d_f Z + r\gamma Z_f \quad (20)$$

The notation used in the above equation has the following meaning

:

$Z_a$  - Impedance determined on the basis of the measured quantities  $U_a$  and  $I_a$  ( $Z_a = U_a/I_a$ ).

$d_f$  - relative distance to the faulted tower expressed in the relation to the total line length and

$Z$  - line impedance determined (in accordance to e.g. [3]) by the following expression:

$$Z = Z_d + (Z_0 - Z_d) \frac{I_0}{I_a} \quad (21)$$

The notation in (21) has the following meaning:

$Z_d$  - positive-sequence impedance for the total line length

$Z_0$  - zero-sequence impedance for the total line length

The impedances  $Z_d$  and  $Z_0$  represent the parameters of the line that can be obtained by measurement in the moment immediately before putting the line in operation. Thus it can be said that the impedance  $Z$  is an a priori known quantity. However, the value of the imaginary part of the impedance  $Z_0$  is often affected by the changeable soil resistivity along the line and it can be different for different sections of the same line. Because of that, in practical situations this impedance can be only approximately taken as a linear function of the line length. By separating the complex equation (20) into its real and imaginary part, we obtain the following two equations

$$\text{Re}\{Z_a\} = d_f R + \text{Re}\{r\gamma Z_f\} \quad (22)$$

$$\text{Im}\{Z_a\} = d_f X + \text{Im}\{r\gamma Z_f\} \quad (23)$$

By dividing (22) with the real part of the impedance  $Z(R)$  and (23) with the imaginary part of the same impedance  $(X)$  we obtain

$$d_r = d_f + \frac{\operatorname{Re}\{r\gamma Z_f\}}{R} \quad (24)$$

$$d_i = d_f + \frac{\operatorname{Im}\{r\gamma Z_f\}}{X} \quad (25)$$

In this system of equations  $d_r$  and  $d_i$  represent rough estimates of the fault distance obtained by using the known  $Z_a$  and  $Z$  and by ignoring the fault impedance ( $Z_f=0$ ). The estimates based on the real and on the imaginary part are mutually different ( $d_r \neq d_i$ ) since according to the quantitative analysis [3,4] the real and the imaginary parts of the impedances  $Z$  and  $Z_f$  in general case are not mutually proportional ( $R/X \neq R_f/X_f$ ).

The only exception is the case of a fault at the end of the line, because only then the impedance  $Z_f$  is negligible ( $Z_f \approx 0$ ). Also, because the ratio between  $\operatorname{Re}\{r\gamma Z_f\}$  and  $\operatorname{Im}\{r\gamma Z_f\}$  changes from tower to tower, it is realistic to assume that the proportion  $R/X = \operatorname{Re}\{r\gamma Z_f\}/\operatorname{Im}\{r\gamma Z_f\}$  in a general case does not exist.

The product of the quantities  $r$ ,  $\gamma$  and  $Z_f$  is unknown and each of these quantities seen separately is unknown. However, the real and the imaginary part of this product are mutually connected by

$$\operatorname{Im}\{r\gamma Z_f\} = \operatorname{Re}\{r\gamma Z_f\} \operatorname{tg} \varphi_f \quad (26)$$

where

$$\varphi_f = \varphi_r + \varphi_\gamma + \varphi_z \quad (27)$$

$\varphi_r$  - phase angle of the line reduction factor  $r$ ,

$\varphi_\gamma$  - phase angle of the coefficient  $\gamma$ , separately considered in [6]

$\varphi_z$  - phase angle of the impedance  $Z_f$ , separately considered in [6] (may be obtained by measurement).

The relations (24), (25) and (26) form a closed system of equations that enables the estimation of the desired distance  $d_f$ . It is determined by the following expression

$$d_f = \frac{Xd_i - Rd_r \operatorname{tg} \varphi_f}{X - R \operatorname{tg} \varphi_f} \quad (28)$$

The effects of numerous line and system parameters considered in Part 3., including the arc resistance [5], are expressed only through  $d_i$ ,  $d_r$  and  $\varphi_f$ .

Regarding the identification of this fault type it is important to mention the following. When a ground fault occurs the transmission line zero-sequence voltage and current can be picked up at the monitoring point. They will not appear in the power system when a non-ground fault (phase-to-phase or three-phase) occurs. Therefore a fault is regarded as non-ground if the signal of zero-sequence voltage does not appear at the monitoring point when a fault occurs. Expressed in another way: if the condition  $d_r \neq d_i$  is satisfied for one phase conductor, we have a single phase-to-ground fault on the line. If this condition is simultaneously satisfied for two phase conductors, we have a double phase-to-ground fault on the line.

Quantitative analysis performed in [5] shows that the value of  $\varphi_\gamma$  varies in very narrow limits along the line that satisfies the condition  $|I_a| > |I_b|$ . This is why the following question arise: are there in practice the lines for which we a priori know that the condition  $|I_a| > |I_b|$  is satisfied always and on the whole length? Lines satisfying this condition exist in power systems and these are the so-called radial lines, serving as a connection between the transmission and the distribution networks, as well as feeding lines in high voltage distribution networks. According to that for this type of lines the angle  $\varphi_\gamma$  can be disregarded so that instead of (27) we can use the following approximation

$$\varphi_f \approx \varphi_r + \varphi_z \quad (29)$$

Based on the quantitative analysis given in [5], it is obtained the conditions under which the angle  $\varphi_f$  can be treated as an a priori known quantity (19). These conditions simultaneously represent conditions for which the application of the developed algorithm gives high accuracy. The favorable circumstance is that the high accuracy is especially desirable for the relays working in such conditions, or in high voltage distribution networks. The lines belonging to these networks are relatively short and because of that the grounding impedance at fault place has relatively great effect on the fault place determining. As a consequence of this the distance relay see the fault

as it if taken place in another zone or even out of line. For overcoming this problem of current engineering practice it is necessary to provide long – distance data transfer. In comparing with this solution the algorithm developed in this paper represents one more economical, more reliable and more practical solution.

## 5. CONCLUSIONS

The paper presents a novel digital algorithm for distance relay including grounding impedance at fault place. The accuracy of this algorithm is high especially for the relays working in high voltage distribution networks. This quality enables us to eliminate the coordination problem appearing in operation these relays without long-distance data transfer.

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